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Bounded Elliptical Modes

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ABSTRACT

The Pekeris waveguide is used to derive an estimation technique for radial wavenumbers for the modal structure of an acoustic field in a given environment. While these estimates are inadequate for coherent processing (such as matched field processing) they are none-the-less useful since they are adequate for waveguide invariant calculations and for environmental province boundary calculations.

PEKERIS WAVEGUIDE

The Pekeris waveguide[1] is an example of a waveguide that can be treated analytically. It is an isovelocity two-fluid model where the sediment is modeled as a fluid half-space. The channel parameters are the water sound speed, c_w, the sediment sound speed, c_s, the channel depth, h, and the ratio of sediment to water density, $b = \rho_s / \rho_w$. The dispersion for this waveguide is given by

$$\tan\left(h\sqrt{\left(\frac{\omega}{c_w}\right)^2 - k_n^2} \right) = -b\sqrt{-\frac{\left(\frac{\omega}{c_w}\right)^2 - k_n^2}{\left(\frac{\omega}{c_s}\right)^2 - k_n^2}} \quad . \tag{1}$$

This is a very useful model because it allows relatively simple calculations, and has many of the features of typical shallow water sites.

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RADIAL WAVENUMBER ESTIMATION

The Pekeris dispersion relationship (1) implies the general bounds

$$\left(\frac{\omega}{c_s}\right)^2 \le k_n^2 \le \left(\frac{\omega}{c_w}\right)^2 \tag{2}$$

on wavenumber as well as an expression for the number of trapped modes[2,3]

$$M \le \frac{1}{2} + \frac{2\,fh}{c_w}\sqrt{1-\left(\frac{c_w}{c_s}\right)^2} \quad . \tag{3}$$

Assuming the usual elliptical functional dependence of the wavenumber on mode-number, we write

$$k_n = \sqrt{\left(\frac{\omega}{c_w}\right)^2 - an^2} \quad , \tag{4}$$

where a is approximated by

$$a = \frac{\left(\frac{\omega}{c_w}\right)^2 - \left(\frac{\omega}{c_s}\right)^2}{M^2} \quad . \tag{5}$$

This arrangement satisfies the bounds expressed in (2).

Equations (3 - 5) allow a quick and dirty approximation of the modal wavenumbers at any point in a large and complicated environment. While such wavenumbers would not be adequate for phase coherent processing such as matched field localization, they would be adequate for tasks such as environmental provincing or calculation of the waveguide invariant[4].

SIMULATION RESULTS

A simple 100 m deep Pekeris environment was constructed with an isovelocity water sound speed of 1500 m/s, a sediment sound speed of 1700 m/s, a water density of 1 g/cc, and a sediment density of 1.5 g/cc. Equations (3 - 5) were applied to this environment to estimate the radial wavenumbers. Other estimates were also calculated for comparison.

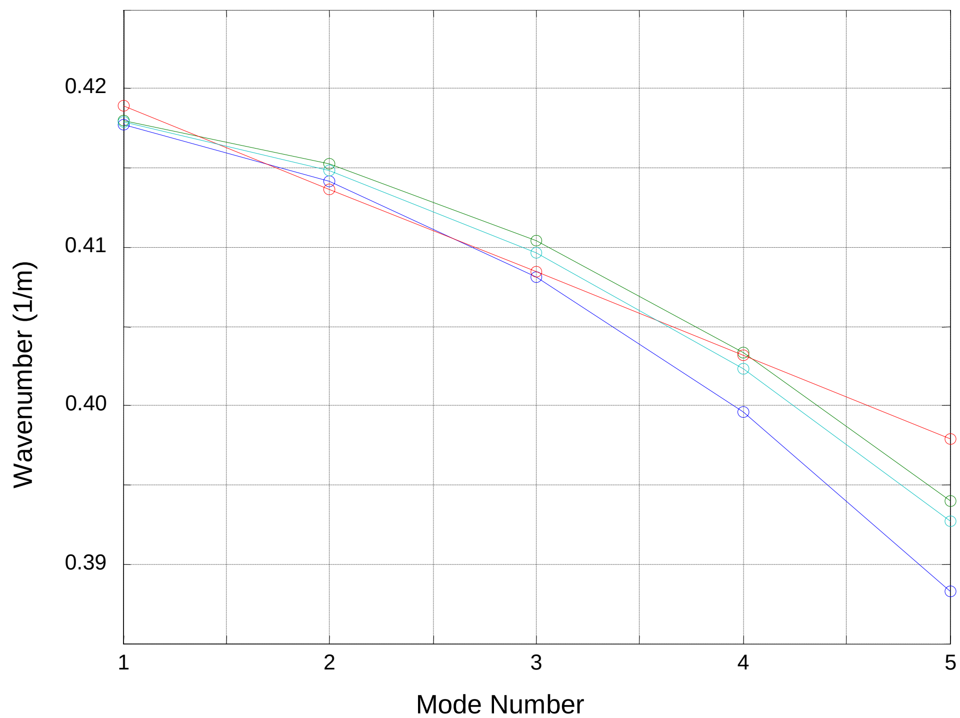

Figure 1- Estimated wavenumbers for Pekeris (green), bounded elliptical (cyan), rigid waveguide (blue), and bounded linear (red).

Figure (1) shows the radial wavenumbers at 100 Hz for several different ways of estimating these wavenumbers. Since the environment is a Pekeris environment, the radial wavenumbers (green) calculated with the Pekeris dispersion (1) are ground truth for this example. The wavenumbers calculated using the rigid waveguide assumption

(blue) are not very close. The wavenumbers calculated using a linear functional dependence on mode number (red) between the bounds given in (2) are not at all satisfactory. But, the wavenumbers calculated using the elliptical functional dependence (cyan) on mode number given by (4) between the bounds given in (2) are reasonably close to (Pekeris) ground truth.

CONCLUSION

It is clear from these results that while the wavenumber estimates are not good enough to perform coherent processing such as matched field processing, they are adequate for waveguide invariant calculations or for defining province boundaries.

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